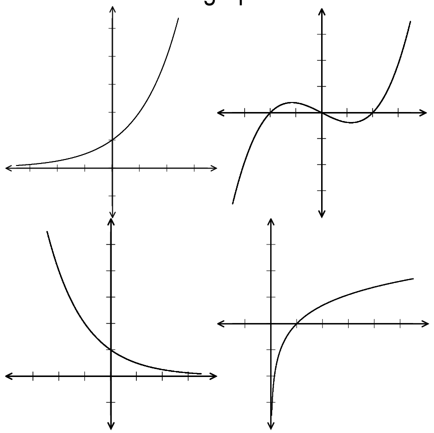
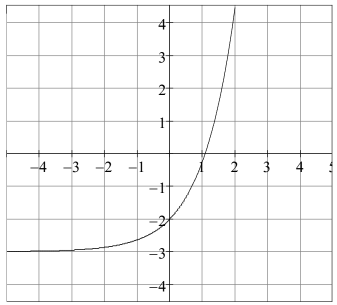
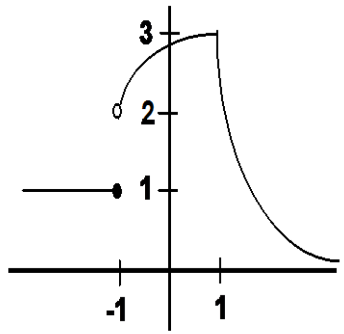
Attachment 02: Tables for CLO, LLO -Assessment Alignment

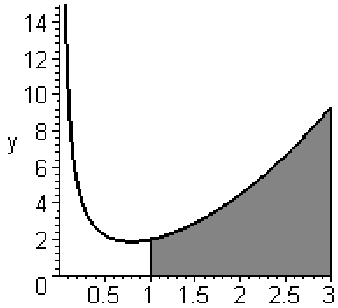
1. Calculus Part (adapted from Dr. Sharifah Sakinah)

LEARNING OUTCOMES	LESSON LEARNING OUTCOMES	ASSESSMENT ITEMS	ASSESSMENT METHOD
	Match the types of function and their graphs.	Match each of the graphs with a function from the list:  a) $f(x) = ax^3 + bx^2 + cx + d$ b) $g(x) = \log_a x$ c) $h(x) = a^x,\ a > 1$ d) $k(x) = a^x, 0 < a < 1$	T1:4
	Produce the value the composite function of two functions.	a) If $f(x) = -4x + 2$ and $g(x) = \sqrt{x-8}$, find $(f \circ g)(12)$ b) If $f(x) = -3x + 4$ and $g(x) = x^2$, find $(g \circ f)(-2)$ c) Given $f(x) = x^2 + 7$ and $g(x) = x - 3$, find $(f \circ g)(x)$ d) Given $f(x) = x - 1$ and $g(x) = x^2 + 2x - 8$, find $(g \circ f)(x)$	T1:4

Apply knowledge and fundamental concepts of Calculus.	Find the inverse of functions	a) Find the inverse of the function $f(x)=\sqrt{x}+1$ b) For each of the following pairs of functions, state whether they are inverses of each other. $f(x)=\dfrac{x+1}{2},\ g(x)=2x-1$ $f(x)=\dfrac{x-1}{2},\ g(x)=2(x+1)$	T1:4
	Sketch the graph of the inverse of a function.	Sketch the graph of the inverse of the function below, on the same grid:  	T1:2
	Simplify the algebraic expressions of exponential forms	Simplify each expression: a) $x^{\sqrt{2}} \cdot x^{\sqrt{5}}$ b) $5^{x-3} \cdot 5^{2x+1}$ c) $9^{a+4b} \cdot 3^{2a+b}$ d) $(7b^{2x})^{3}$ e) $(a^{\sqrt{6}})^{\sqrt{2}}$ f) $(m^{2x+5})^{3x}$	T1:6

	Solve equations involving exponentials and logarithms.	Solve each equation: a) $4^{2x+3} = 64^{x-1}$ b) $3^{5-x} = 27^{2x}$ c) $\log_8(x_2 - 2x) = \log_8 3$	T1:3
	Evaluate forms involving exponentials and logarithms.	Evaluate the logarithms. a) $\ln e^{-7}$ b) $\log\left(\frac{1}{1000}\right)$ c) $\log_3 27$ d) $\ln 15$ e) $\log_3 6$	T1:5
	Find limits of polynomials and rational functions.	Discover each of the following limits (show your work): a) $\lim\limits_{x\to 3} 4\pi$ b) $\lim\limits_{x\to 3} \frac{3-x}{x^2+2x-15}$ c) $\lim\limits_{x\to 1^+} \frac{x}{x-1}$ d) $\lim\limits_{x\to 1} \frac{x}{x-1}$ e) $\lim\limits_{x\to -\infty} \frac{3x^2-1}{2-3x}$	MT1:5
	Determine the continuity of a step-wise function at a point.	Consider the following function: $f(x) = \begin{cases} x^2, & \text{if} \quad x \geq 0 \\ x-2, & \text{if} \quad x < 0 \end{cases}$ a) Find $\lim\limits_{x\to 0^-} f(x)$ b) Find $\lim\limits_{x\to 0^+} f(x)$	MT2:5

		c) Find $\lim_{x \to 2} f(x)$ (note that x approaches two, not zero) d) Is the function continuous at x = 0	
	Determine the continuity and differentiability of function at some points based on a given graph.	 The picture on the above shows the graph of a certain function. Based on that graph, answer the questions: a) $\lim_{x \to -1^-} f(x)$ b) $\lim_{x \to -1^+} f(x)$ c) $\lim_{x \to 1} f(x)$ d) $\lim_{x \to 0} f(x)$ e) Is the function continuous at x = -1? f) Is the function continuous at x = 1? g) Is the function differentiable at x = -1? h) Is the function differentiable at x = 1? i) Is f'(0) positive, negative, or zero? k) What is f'(-2) ?	MT3:5
	Compute the derivatives of polynomial, exponential and rational functions	Compute the derivatives of the following functions and simplify your answers as much as possible:	MT4:5

		a) $f(x) = \dfrac{x^2+3e^x}{2e^x-x}$ b) $f(x) = x^3e^{2x}$ c) $y = 3x^5 - 4\sqrt{x} - \dfrac{2}{x^2} + \dfrac{1}{\sqrt[4]{x}}$ d) $y = (3x^2-5x+1)(2\sqrt{x}-5)$	
	Find infinite and definite integrals of polynomial and exponential functions with the use of techniques of integration.	Integrate the following and give exact values: a) $\int_{-\ln 2}^{0} e^{2x}\, dx$ b) $\int (2-x)^{3/5} dx$	MT5:5
	Find the area of space bounded by curves with the use of integration techniques.	Find the area of the shaded region.  $f(x) = x^2 + \dfrac{1}{x}$	MT6:5
	Solve word problems involving natural exponential functions.	The number of bacteria N in a culture is given by the model N= 240 e ^{kt} where t is time in hours and t=0 corresponds to the time when N = 240. When t = 10, the bacteria count is N = 320. How long does it take for the population of bacteria to reach N = 720 ?	A1:1:1

Solve real-life application problems using suitable techniques in Calculus or Numerical Methods			
	Solve word problems involving natural exponential functions.	The estimated population of the city of Peru is given by the equation $P = 650,000e^{.07t}$ where t is the number of years from now. a) What will the population be in 10 years? b) How long will it take the population to reach 1 million?	MT7:10
	Use the derivative concept to *find the volume of the largest open box made from a square cardboard.*	Find the volume of the largest open box that can be made from a piece of cardboard 24 inches square by cutting equal squares from the corners and turning up the sides.	A1:2:2
	Use derivative concept to determine the velocity and the acceleration of a dropped cannonball.	A cannonball is dropped from a height of 179 feet above the ground. The height of the ball after *t* seconds is given by $s(t) = 179 - 16t^2$. Find the velocity and acceleration functions. *Compute* the instantaneous velocity [s'(c)] of the cannonball at 2 seconds and the acceleration of the ball at 2 seconds.	A1:3:2
	Use derivative concept to compute the velocity of a car given the distance from the origin ***s*** as a function of time ***t.***	After ***t*** hours a car is a distance ***s*** miles from its starting point, where $$s(t) = 60t + \frac{100}{t+3}$$ Compute its velocity after 2 hours.	MT8:10
	Solve an initial value word problem involving first order linear ordinary differential equations.	A model for the spread of a rumour is that the rate of spread is proportional to the product of the fraction of the population who have heard the rumour and the fraction who have not heard the rumour. The differential equation that models this is $$\frac{dy}{dt} = ry(1-y)$$ Where y denotes the fraction of the population who has heard the rumour and r denotes the rate at which the rumour spread. The solution of this differential equation is, $$y = \frac{y_0}{y_0 + (1-y_0)e^{-rt}}$$ (Be sure to show all the details when you solve your differential equation) Then, assume a small town has 2000 peoples. At 8 AM, 80 people have heard a rumour. By noon, half the town has heard it. We would like to know at what time 90% of the population will have heard the rumour.	A1:4:3

		After showing some work (be sure to show this work when you do this), 90% of the population will hear the rumour by 3:36 PM.	
	Solve an initial value word problem involving first order linear ordinary differential equations	A certain radioactive material is known to decay at a rate proportional to the amount present. If initially there is 50 milligrams of the material present and after two hours it is observed that the material has lost 10 percent of its original mass, find (a) an expression for the mass of the material remaining at any time t, (b) the mass of the material after four hours, and (c) the time at which the material has decayed to one half of its initial mass.	A1:5:2
	Using integral concept, find the time of a ball reach the maximum height when it is thrown upward.	A ball is thrown straight upward from ground level on a planet whose acceleration due to gravity is constant at $-48\ \text{ft/s}^2$. After 2 seconds, ball's velocity is 30 ft/s. Use calculus to compute the time at which the ball reaches its maximum height.	MT9:10

Legend:
For Assessment Method column, for example, A1:5:2 means "the item will be given as Assignment 1, problem number 5, and with score of 2.

2. Numerical Methods Part

No.	Course Learning Outcomes	Lesson Learning Outcomes		Assessment Items	
		No.	Items	No.	Items
1	Apply knowledge and fundamental concepts Numerical Methods.	1.1	Find a general formula for the n degree Taylor polynomial of a natural exponential function at the point of approximation a=0.	1.1.1	Produce a general formula for the **n** degree Taylor polynomial for e^{-x} about *0*. How large ***n*** should be chosen in the Taylor polynomial of degree ***n***, $P_n(x)$ such that $\lvert e^{-x}-P_n(x)\rvert \le 10^{-5}$ for $-1 \le x \le 1$? (FE1a:3).
		1.2	Find the minimum degree of Taylor polynomial such that a given error of approximation is satisfied.		
		1.3	Use nested multiplication technique to show how to evaluate a multi exponential function efficiently.	1.3.1	Show how to evaluate the function $f(x)=2e^{4x}-e^{3x}+5e^{x}+1$ efficiently (FE1b:1).
		1.4	Use the technique of reformulating algebraic expression to avoid loss-of-significance error.	1.4.1	Use the technique of algebraic expression reformulation to avoid the loss-of-significance error in solving $x^2-26x+1=0$ (FE1c:1).
		1.5	Given three sets of points, find first order and second order divided differences.	1.5.1	Given the data (0.1,0.2), (0.2,0,24), and (0.3,0.3). Find $f[x_0,x_1]$ and $f[x_0,x_1,x_2]$. Then calculate $P_1(0.15)$ and $P_2(0.15)$, which are the linear and quadratic interpolates evaluated at $x=0.15$ (FE2: 5).
		1.6	Calculate the linear and quadratic interpolates evaluated at a given point.		
		1.7	Given a function and three set of points of the domain, find the errors of the quadratic interpolates evaluated at the points	1.7.1	Let $f(x)=1/(1+x)$ and let $x_0=0$, $x_1=1$, and $x_2=2$. Calculate the divided differences $f[x_0,x_1]$ and $f[x_0,x_1,x_2]$. Using the divided differences, give the quadratic polynomial $P_2(x)$ that interpolates $f(x)$ at the given node points. Find the errors of interpolation on that points. (FE3: 5).

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		1.8	Compare the errors of the approximate values produced by the use of forward, central, and backward difference methods for two step sizes for a given function.	1.8.1	Using forward, central, and backward difference methods, find the numerical derivative $D_h f(x)$ at $x=0$ for $f(x) = e^{-x}$. Use the step size $h=0.1$ and $h=0.2$. Compare the approximate values in terms of their errors across the methods and the step sizes used. (FE4:6)
2	Solve problems particularly in computer science with appropriate and high-level programming language or tools.	2.1	With utilization MATLAB®, solve a equation by using bisection, Newton's, and secant methods	2.1.1	Use the Bisection, Newton's and Secant Methods to find the solution (to at least 8 significant figures) of the equation ***sin(x) = cos(2x²)***.over the interval $[0,\pi]$. For Newton's method, try with several different initial guesses including $x_0 = 1$. For the Secant method use the same initial values as for bisection, as well as other appropriate values. Comment on the results, comparing the effectiveness of each algorithm for this problem. In particular, compare the number of iterations required by each method to find the answer to a specified accuracy and see if the behavior you observe tallies with theoretical predictions where they have been given, in particular the order of convergence. Provide a careful explanation of your observations. Hint: draw a graph of the equation to understand the behavior (A2:10). *Note: every group of students will be given a different set of equation.*
		2.2	Compare the relative effectiveness of the methods in solving the equation		
		2.3	Explain the observation carefully and provide a written report on those.		
		2.4	Given the equation of a curve, compare the accuracy of approximate values produced by using trapezoidal and Simpson's rule for the area under the curve over an interval.	2.4.1	Use the trapezoidal rule and Simpson's rule with $n=10$ to find approximate values of the area under the curve $y = e^{-x^2}$ over the interval $[0,2]$. a) Present all necessary computations and results in 8 (eight) digit significant figures. b) Compare the approximate values in terms of their accuracy if the exact value is 0.82643816. (T2:10).
3	Solve real-life application problems using suitable techniques in Calculus or Numerical Methods	3.1	Solve a word-problem in initial value problems of first-order differential equation using Euler's, Heun's, and Classical Runge-Kutta methods	3.1.1	Consider the motion of a particle of mass m falling vertically under the earth's gravitational field, and suppose the downward motion is opposed by a frictional force $p(v)$ dependent on the velocity $v(t)$ of the particle. Then the velocity satisfies the equation $mv'(t)=-mg+p(v), t\geq 0, v(0)$ given. Let $m=1$ kg, $g=9.8$ m/sec2, and $v(0)=0$. Find the velocity of the particle for the period of the first 3 seconds using step size h = 1 second, provided that $p(v)=-0.1v$, which is positive for a falling body by using: Euler's method, Heun's method, and Classical Runge-Kutta method (FE5:9).

Legend: For Assessment Items column, for example, FE1a:3 means "the item will be given as Final Exam problem 1a and with score of 3.